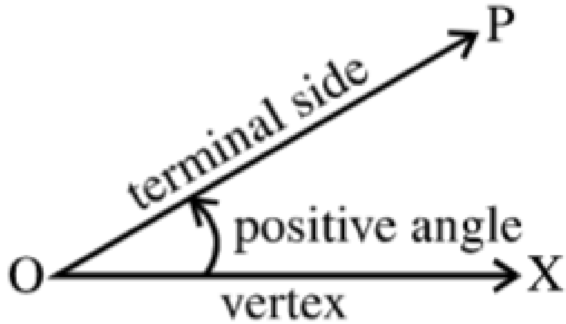
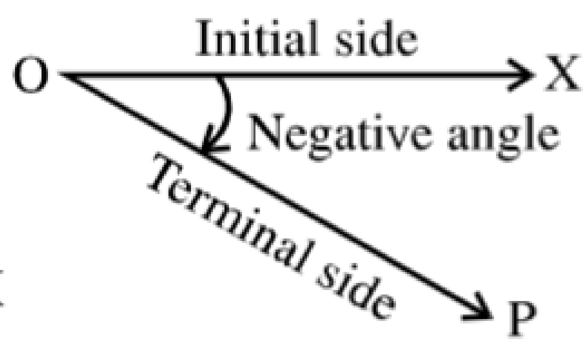
TRIGONOMETRIC FUNCTIONS

Trigonometry: The word trigonometry is derived from two Greek words, trigonon (triangle) and metron (to measure). Therefore, literal meaning of Trigonometry is "to measure a triangle". But now a days it is defined as that branch of mathematics which deals with angles, whether of a triangle or any other figure.

An Angle: In trigonometry an angle is defined as the amount of rotation made by a straight line from one position to another position about a point. If the initial side OX moves in anticlockwise direction to the terminal side OP from the vertex O, then





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the angle XOP as shown is called a positive angle. But if, on the other hand, the initial side OX moves in the clockwise direction as shown in figure, then XOP, traced out in this manner, is called a negative angle.

Measurement of Angles: In general, the angles are measured in degrees or radians which are defined as follows:

Degrees: A right angle is divided into 90 equal parts and each part is called a degree. Thus a right angle is equal to 90 degrees. One degree is denoted by $1°$. A degree is divided into sixty equal parts and each part is called a minute and is denoted by $1'$: A minute is divided into sixty equal parts and each part is called a second and is denoted by $1''$.

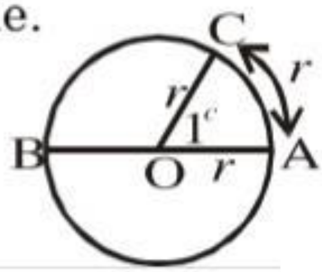
Thus we have,

1 right angle = $90°$ (read as 90 degrees)

$1° = 60'$ (read as 60 minutes)

$1' = 60''$ (read as 60 seconds)

Radians: A radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.



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In this figure OA = OC = arc AC = r = radius of the circle, then measurement of $\angle$AOC is one radian and is denoted by 1^c. Thus $\angle AOC = 1^c$

A Constant Number π: The ratio of the circumference to the diameter of a circle is always equal to a constant and this constant is denoted by the Greek letter π.

Thus π = Circumference/diameter.

$\therefore$ If r is the radius of a circle, then its circumference = $2\pi r$.

The constant π is an irrational number and its approximate value is taken as $\frac{22}{7}$.

Relation between an Arc and an Angle: If s is the length of an arc of a circle of radius r, then the angle θ (in radians) subtended by this arc at the centre of the circle is given by

$\theta = s/r$ or $s = r\theta$

i.e., arc = radius × angle in radians.

Thus, from the above figure.

$$\angle AOB = \frac{\text{arc ACB}}{r} \text{ (in radians)} = \frac{\pi r}{r} = \pi \text{ radians}$$

Hence, we have, π radians = 180° = 2 right angles.

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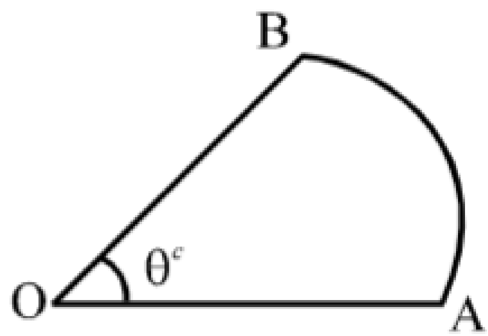
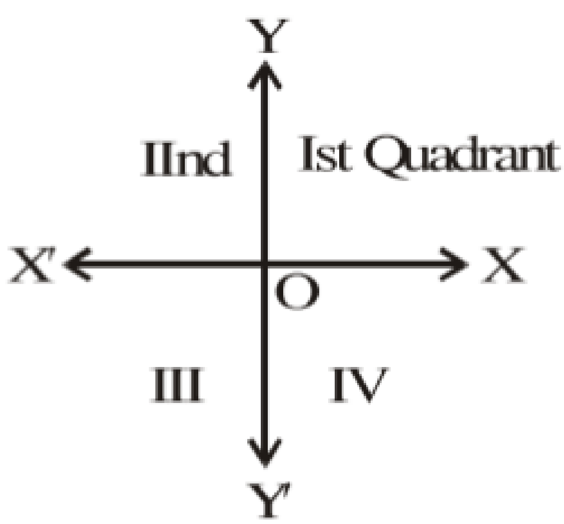
or 1 radian = $\frac{180}{\pi}$ degrees = $\frac{180}{22} \times 7$ degrees

= $57°17'44.8''$ (Appr.)

Sectorial Area: Let OAB be a sector having central angle θ^c and radius r. Then area of the sector OAB is given by $\frac{1}{2} r^2\theta$.



Quadrants: Let XOX′ and YOY′ be two mutually perpendicular lines in any plane. These lines divide the plane into four parts and each one of them is called quadrant.



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(*i*) The region XOY is called First Quadrant.

(*ii*) The region YOX′ is called the Second Quadrant.

(*iii*) The region X′OY′ is called Third Quadrant.

(*iv*) The region Y′OX is called the Fourth Quadrant.

Radians measure of Some Common angles:

Angle in Degrees	30°	45°	60°	90°	180°	270°	360°	540°	720°
Angle in Radian	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π	3π	4π

Relation between the three systems of measurement of an angle: The three systems of the measurement of an angle are related by formula

$$\pi \text{ radian} = 180° = 200^g.$$

Theorem: Circumference of a cirlce bears a constant ratio to its diameter.

$$\text{i.e.,} \quad \frac{\text{Circumference}}{\text{Diameter}} = \pi = 22/7$$

$$= \frac{355}{113} \text{ nearly}$$

(In more accurately)

$= 3.1416$ nearly.

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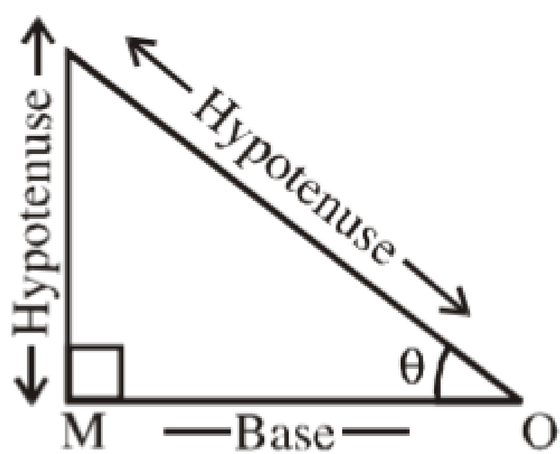
Theorem: The angle, in radians, subtended by an arc of the circle at the centre $= \frac{\text{arc}}{\text{radius}}$

i.e., θ = angle subtended at the centre.

$$= \frac{\text{Length of arc}}{\text{Radius of the circle}}$$

$$= \frac{AC}{AB}$$

Trigonometric Ratios or functions: Let the revolving line OP start from its initial position OX and trace out an angle XOP = θ in any one of the four quadrants. From P draw PM perpendicular to X′OX.



Now in the right angled triangle POM, if θ is the angle of reference, then MP, the side opposite to θ

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is called perpendicular, OP the side opposite to right angle is called the hypotenuse and OM, the third side is called base.

(i) $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{MP}{OP}$

(ii) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OM}{OP}$

(iii) $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{MP}{OM}$

(iv) $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{OM}{MP}$

(v) $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{OP}{OM}$

(vi) $\operatorname{cosec}\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{OP}{MP}$

Remember

(a) The student should not commit mistake by regarding $\sin\theta$ as $\sin x\,\theta$. $\sin\theta$ is correctly read as the sine of the angle θ.

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(*b*) The student should note that $(\sin\theta)^n$ is written as $\sin^n\theta$ if $n \neq -1$.

For Example,

$$(\sin\theta)^2 = \sin^2\theta$$
$$(\sin\theta)^3 = \sin^3\theta$$

but $(\sin\theta)^{-1} \neq \sin^{-1}\theta$; Also, $\sin^{-1}\theta \neq \dfrac{1}{\sin\theta}$

Relationship between trigonometric functions:

(*i*) Reciprocal Relation: The following relations are obvious from the definition of t-ratios.

(*a*) $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$ and $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

(*b*) $\cos\theta = \dfrac{1}{\sec\theta}$ and $\sec\theta = \dfrac{1}{\cos\theta}$

(*c*) $\tan\theta = \dfrac{1}{\cot\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$

(*d*) $\sin\theta \times \operatorname{cosec}\theta = 1$

(*e*) $\cos\theta \times \sec\theta = 1$

(*f*) $\tan\theta \times \cot\theta = 1$

(*ii*) Quotient Relation

(*a*) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

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(b) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(iii) Square Relations

(a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\sec^2\theta - \tan^2\theta = 1$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1$

Trigonometric Identities: Trigonometric Identities is a statement of equality between the expressions and is true for all values of the variable involved. In order to prove them, we give below some of the methods to be used:

Method I: Simplify L.H.S. or R.H.S. which ever is complicated and prove it to be equal to the other side.

Illustrations: Prove that

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

Soln:

$$\begin{aligned}
\text{L.H.S.} &= \sin^4\theta + \cos^4\theta \\
&= (\sin^2\theta)^2 + (\cos^2\theta)^2 \\
&= (\sin^2\theta + \cos^2\theta)^2 \\
&\qquad - 2\sin^2\theta\cos^2\theta \\
&[\because a^2 + b^2 = (a+b)^2 - 2ab] \\
&= (1)^2 - 2\sin^2\theta\cos^2\theta \\
&= 1 - 2\sin^2\theta\cos^2\theta \\
&= \text{R.H.S.}
\end{aligned}$$

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Method II: Change all the trigonometrical ratios in terms of the sines and cosines of the angles.

Illustration: Prove that

$$(\tan \alpha + \cot \alpha)^2 = \sec^2 \alpha \operatorname{cosec}^2 \alpha$$

Soln:

$$\begin{aligned}
\text{L.H.S.} &= (\tan \alpha + \cot \alpha)^2 \\
&= \left(\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha}\right)^2 \\
&= \left(\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha . \sin \alpha}\right)^2 \\
&= \left(\frac{1}{\cos \alpha \sin \alpha}\right)^2 \\
&= \frac{1}{\cos^2 \alpha} \cdot \frac{1}{\sin^2 \alpha} \\
&= \sec^2 \alpha \, . \operatorname{cosec}^2 \alpha \\
&= \text{R.H.S.}
\end{aligned}$$

Method III: Simplify L.H.S. and R.H.S. both, if both are complicated and then prove L.H.S. = R.H.S.

Illustration: Prove that

$\sin^2 A \tan A + \cos^2 A \cot A + 2 \sin A \cos A$
$= \tan A + \cot A$

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Soln: L.H.S. $= \sin^2 A \tan A + \cos^2 A \cot A + 2 \sin A \cos A$

$$= \sin^2 A \frac{\sin A}{\cos A} + \cos^2 A \frac{\cos A}{\sin A} + 2 \sin A \cos A$$

$$= \frac{\sin^3 A}{\cos A} + \frac{\cos^3 A}{\sin A} + 2 \sin A \cos A$$

$$= \frac{\sin^4 A + \cos^4 A + 2\sin^2 A \cos^2 A}{\cos A \sin A}$$

$$= \frac{(\sin^2 A + \cos^2 A)^2}{\cos A \sin A}$$

$[\because (a + b)^2 = a^2 + b^2 + 2ab]$

$$= \frac{1}{\cos A \sin A}$$

Now R.H.S. $= \tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$$= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$$

$$= \frac{1}{\cos A \sin A}$$

Hence, L.H.S. = R.H.S. proved.

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Method IV: From the identity to be proved, obtain another identity which is obviously true by cross-multiplication or by transpositioning.

Illustration: Prove that

$$\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$$

Soln.: $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}$

$$=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$$

If transposing

$$\frac{1}{\sec A+\tan A}+\frac{1}{\sec A-\tan A}=\frac{1}{\cos A}+\frac{1}{\cos A}$$

$$\Rightarrow \frac{\sec A-\tan A+\sec A+\tan A}{(\sec A+\tan A)(\sec A-\tan A)}=\frac{2}{\cos A}$$

$$\Rightarrow \frac{2\sec A}{\sec^2 A-\tan^2 A}=\frac{2}{\cos A}$$

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$$\Rightarrow \frac{2 \sec A}{1} = \frac{2}{\cos A}$$

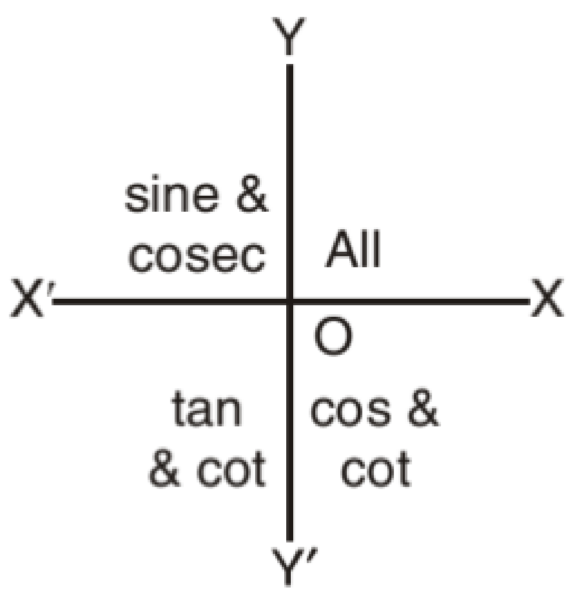
$$\Rightarrow \frac{2}{\cos A} = \frac{2}{\cos A} \text{ which is true.}$$

Sign of trigonometric ratios in different quadrants:

(i) In first quadrant all the t-ratios are positive

(ii) In second quadrant sin θ and cosec θ are +ve and other t-ratios are –ve.

(iii) In third quadrant, tan θ and cot θ are +ve and other t-ratios are –ve.



(iv) In fourth quadrant, cos θ and sec θ are +ve and other t-ratios are –ve.

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Limits to the values of t-ratios:

(*i*) sin θ and cos θ can not be numerically greater than 1.

(*ii*) sec θ and cosec θ can not be numerically less than 1.

(*iii*) tan θ and cot θ can have any numerical values.

Trigonometric Ratios of Standard Angles

	0°	15°	18°	22.5°	30°	36°	45°	60°	67.5°	90°
sin	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{5}-1}{4}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{\sqrt{2}+1}}{\sqrt{(2\sqrt{2})}}$	1
cos	1	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{\sqrt{\sqrt{2}+1}}{(2\sqrt{2})}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{5}+1}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	0
tan	0	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\frac{1}{\sqrt{3}}$	$\sqrt{5-2\sqrt{5}}$	1	$\sqrt{3}$	$\sqrt{2}+1$	not defined

Trigonometric Ratios of Allied Angles

Allied angle: Two angles are said to be allied when their sum or difference is a multiple of 90°. The angle –θ, 90° ± θ, 180° ± θ, etc. are angles allied to the angle θ.

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t-ratio	$-\theta$	$90°-\theta$	$90°+\theta$	$180°-\theta$	$180°+\theta$	$270°-\theta$	$270°+\theta$	$360°-\theta$	$360°+\theta$
$\sin\theta$	$-\sin\theta$	$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$
$\cos\theta$	$\cos\theta$	$\sin\theta$	$-\sin\theta$	$-\cos\theta$	$-\cos\theta$	$-\sin\theta$	$\sin\theta$	$\cos\theta$	$\cos\theta$
$\tan\theta$	$-\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$	$\cot\theta$	$-\cot\theta$	$-\tan\theta$	$\tan\theta$
$\cot\theta$	$-\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$	$\tan\theta$	$-\tan\theta$	$-\cot\theta$	$\cot\theta$
$\sec\theta$	$\sec\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$	$\sec\theta$	$\sec\theta$
$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$\sec\theta$	$\sec\theta$	$\text{cosec}\,\theta$	$-\text{cosec}\,\theta$	$-\sec\theta$	$-\sec\theta$	$-\text{cosec}\,\theta$	$\text{cosec}\,\theta$

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The above results may be obtained by the following rules:

Rule 1: The trigonometrical ratio of $90° \pm \theta$, $270° \pm \theta$ is changed *i.e.*, $\sin \to \cos$, $\cos \to \sin$, $\tan \to \cot$. The positive or negative sign depends on quadrant.

For example, to get a value of $\sin(270° + \theta)$,sin is changed to cos, and since angle $270° + \theta$ is in the 4th quadrant in which sign of sin is –ve.

$\therefore \sin(270° + \theta) = -\cos\theta$.

Rule 2: The trigonometrical ratio of $180° \pm \theta$, $360° \pm \theta$ is not changed and the +ve or –ve sign depend on the quadrant rule.

For example, to get the value of $\cos(180° - \theta)$, cos is not changed, and since angle $180° - \theta$ is in the second quadrant in which the sign of cos is –ve.

$\therefore \cos(180° - \theta) = -\cos\theta$.

Hence, in general consider the angles $\frac{1}{2}n\pi + \theta$ and $\frac{1}{2}n\pi - \theta$, $n \in I$, then

(*i*) assuming that $0 < \theta < 90°$, the result has the plus or the minus sign according as the given function is positive or negative in that quadrant.

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(*ii*) If n is even, the result contains the same trigonometric function as the given expression, but if n is odd, the result contains the corresponding co-function, *i.e.*, sine becomes cosine, tangent becomes cotangent, secant becomes cosecant and vice-versa.

For example, consider $\cos(450° - \theta)$. We have $450° = 5 \times 90°$, so $450° - \theta$ is a first quadrant angle, and n is odd, so $\cos(450° - \theta) = \sin\theta$

Also, this can be found as

$\cos(450° - \theta) = \cos(360° + 90° - \theta) = \cos(90° - \theta)$
$= \sin\theta$

Some interesting results about allied angles:

(1) $\cos n\pi = (-1)^n$, $\sin n\pi = 0$

(2) $\cos(n\pi + \theta) = (-1)^n \cos\theta$

$\sin(n\pi + \theta) = (-1)^n \sin\theta$

(3) $$\cos\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n+1}{2}} \sin\theta \text{ if } n \text{ is odd}$$

$$= (-1)^{n/2} \cos\theta \text{ if } n \text{ is even.}$$

(4) $$\sin\left(\frac{n\pi}{2} + \theta\right) = (-1)^{\frac{n-1}{2}} \cos\theta \text{ if n is odd}$$

$$= (-1)^{n/2} \sin\theta \text{ if } n \text{ is even.}$$

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n	$\frac{n\pi}{2}+\theta$	$\cos\left(\frac{n\pi}{2}+\theta\right)$
1	$\frac{\pi}{2}+\theta$	$-\sin\theta$
2	$\pi+\theta$	$-\cos\theta$
3	$\frac{3\pi}{2}+\theta$	$\sin\theta$
4	$2\pi+\theta$	$\cos\theta$
–	–	–

Periodicity and Graphical representation of trigonometric functions: If a function $f(x) = f(x + \alpha)$, where α is the least positive constant, then $f(x)$ is called the periodic function of x and α is called its period.

Theorem: $\sin\theta$, $\cos\theta$, $\sec\theta$, $\operatorname{cosec}\theta$ are periodic functions with period 2π whereas $\tan\theta$ and $\cot\theta$ are periodic functions with period π.

i.e., $\sin(\theta + 2\pi) = \sin\theta$

$\cos(\theta + 2\pi) = \cos\theta$

$\sec(\theta + 2\pi) = \sec\theta$

$\operatorname{cosec}(\theta + 2\pi) = \operatorname{cosec}\theta$

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i.e., sinθ, cosθ, secθ, cosecθ remains unchanged, when θ is increased by the least positive constant 2π.

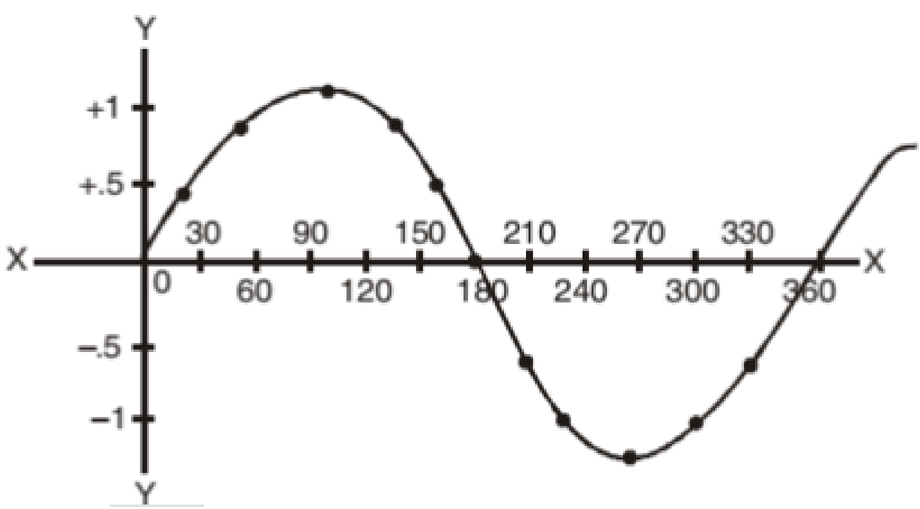
Hence, sinθ, cosθ, secθ, cosecθ are periodic function with period 2π...

Again tan (θ + π) = tan θ; cot (θ + π) = cot θ

i.e., tanθ, cotθ remain unchanged when θ is changed to θ + π, where π is the least positive constant and is called period.

Graph of sin *x*. Table of Values

x	$y = \sin x$
0°	0
30°	.5
60°	.87
90°	1
120°	.87
150°	.5
180°	0
210°	–.5
240°	–.87
270°	–1
300°	–.87
330°	–.5
360°	0



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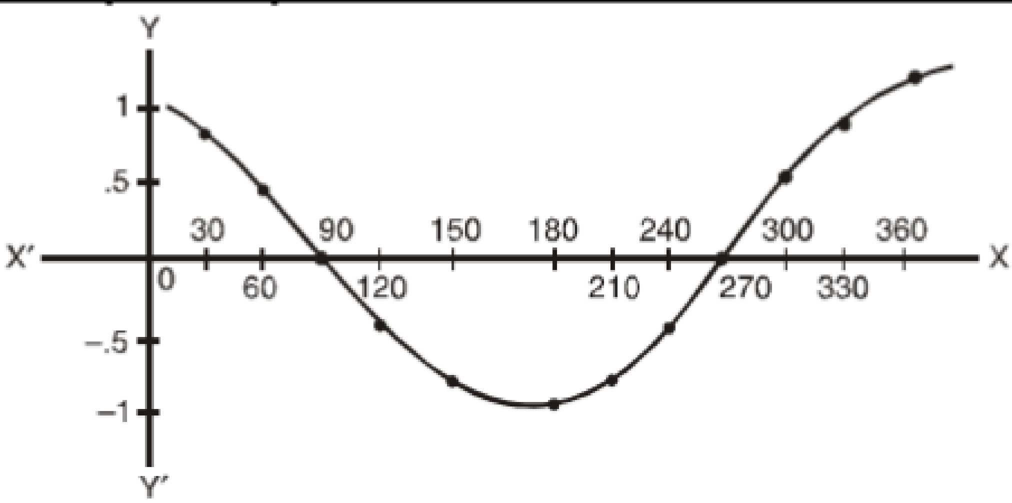
Graph of cos x : Table of Values

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$	1	.87	.5	0	–.5	–.87	–1	–.87	–.5	0	.5	.87	1



Graph of tan x : Table of Values

x	0°	30°	60°	90–0°	90+0°	120°	150°	180°	210°	240°	270–0°	270+0°	300°	330°	360°
$y = \tan x$	0	.58	1.73	+ ∞	– ∞	–1.73	–.58	0	.58	1.73	+ ∞	– ∞	–1.73	–.58	0

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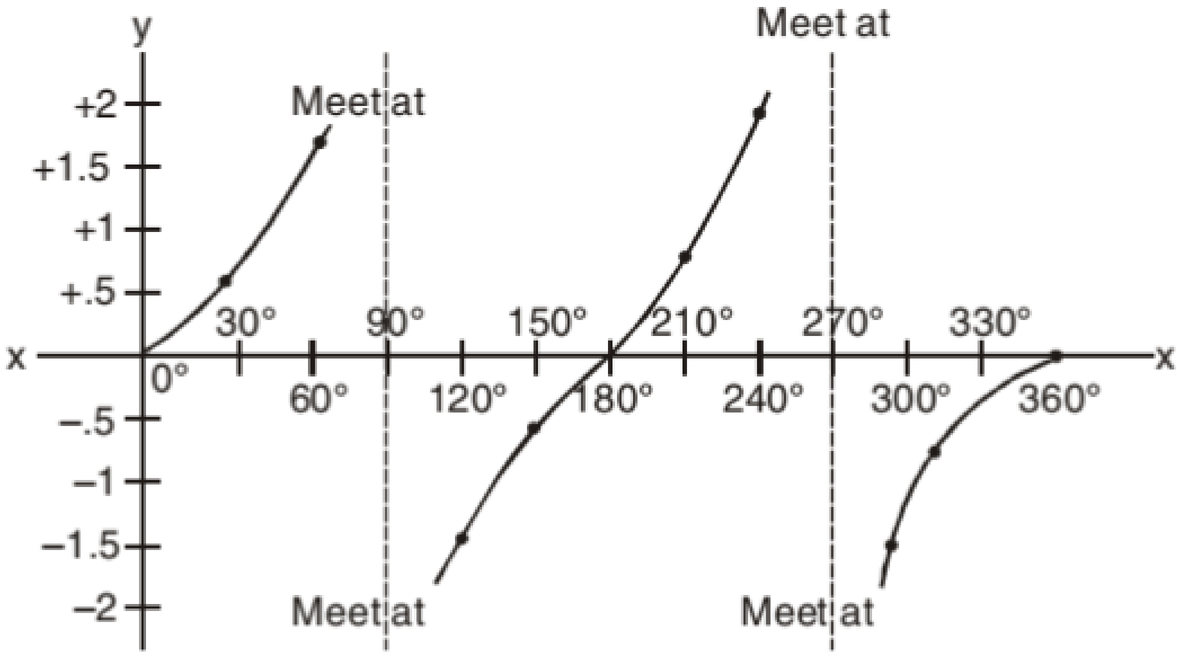

Graph of cot x : Table of Values

x	$y = \cot x$
0° + 0°	∞
30°	1.73
60°	.58
90°	0
120°	–.58
150°	–1.73
180° – 0°	– ∞
180° + 0°	+ ∞
210°	1.73
240°	.58
270°	0
300°	–.58
330°	–1.73
360°–0°	–∞

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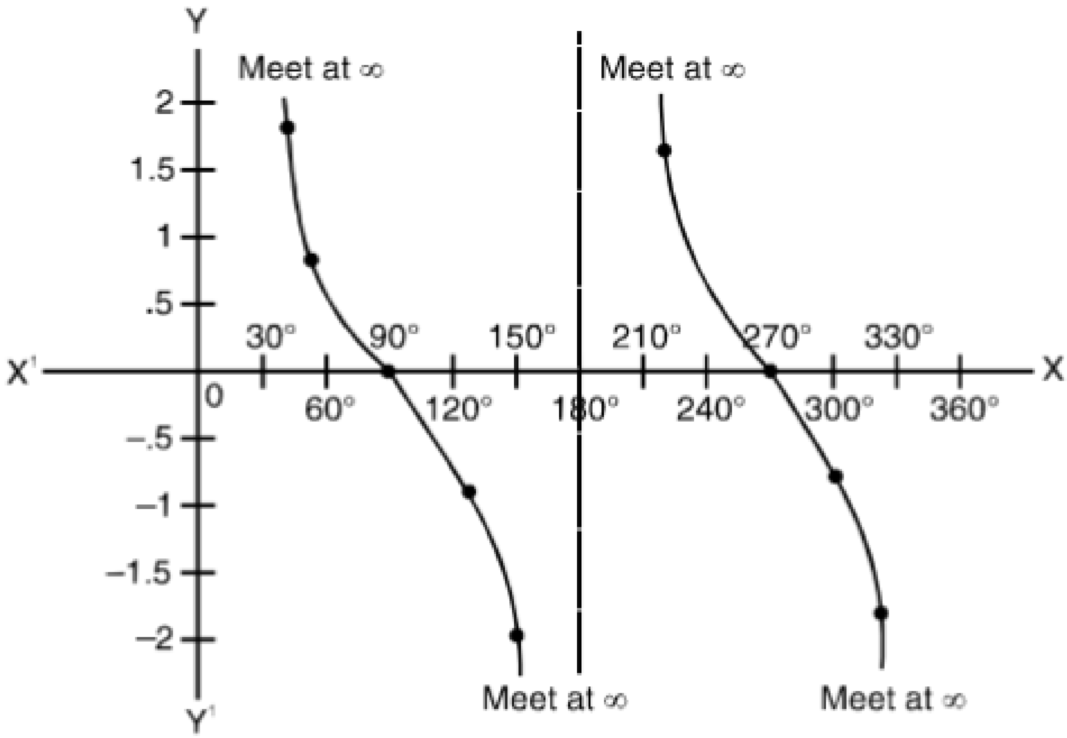

Graph of sec x : Table of Values

x	$y = \sec x$
0°	1
30°	1.15
60°	2
90° – 0°	+ ∞
90° + 0°	– ∞
120°	–2
150°	– 1.15
180°	–1
210°	–1.15
240°	–2
270° – 0°	–∞
270° + 0°	+ ∞
300°	2
330°	1.15
360°	1

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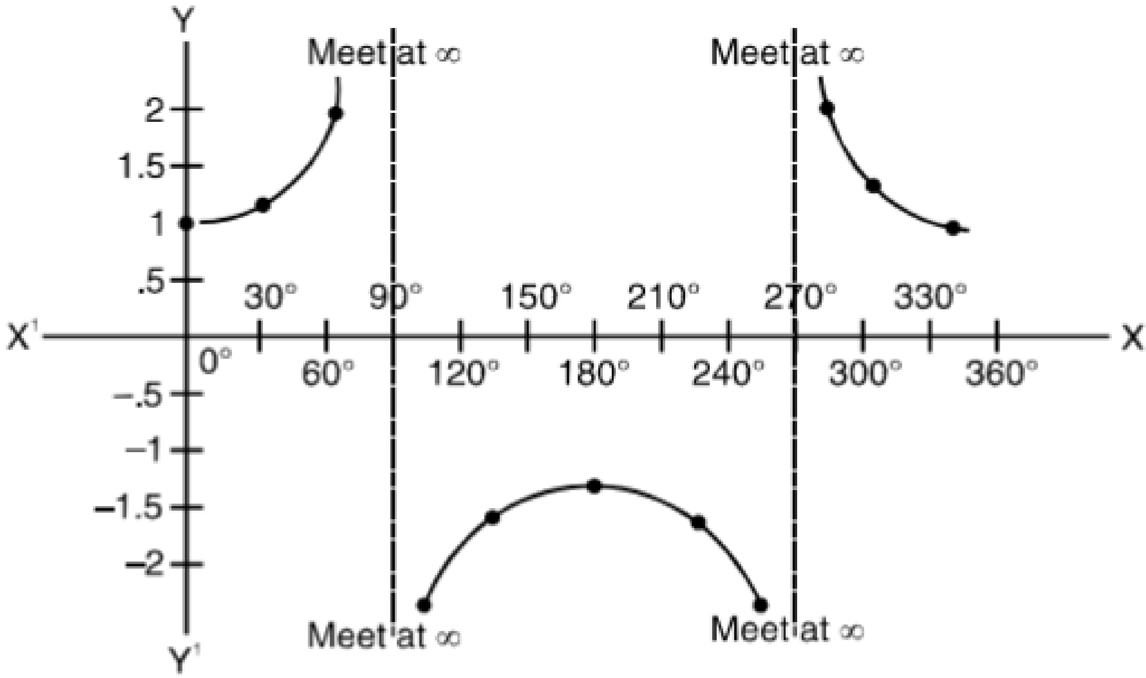

Graph of cosec x : Table of values

x	0° + 0°	30°	60°	90°	120°	150°	180°– 0°	180° + 0°	210°	240°	270°	300°	330°	360° – 0°
$y = \text{cosec } x$	+ ∞	2	1.15	1	1.15	2	+ ∞	– ∞	–2	–1.15	–1	–1.15	–2	–∞

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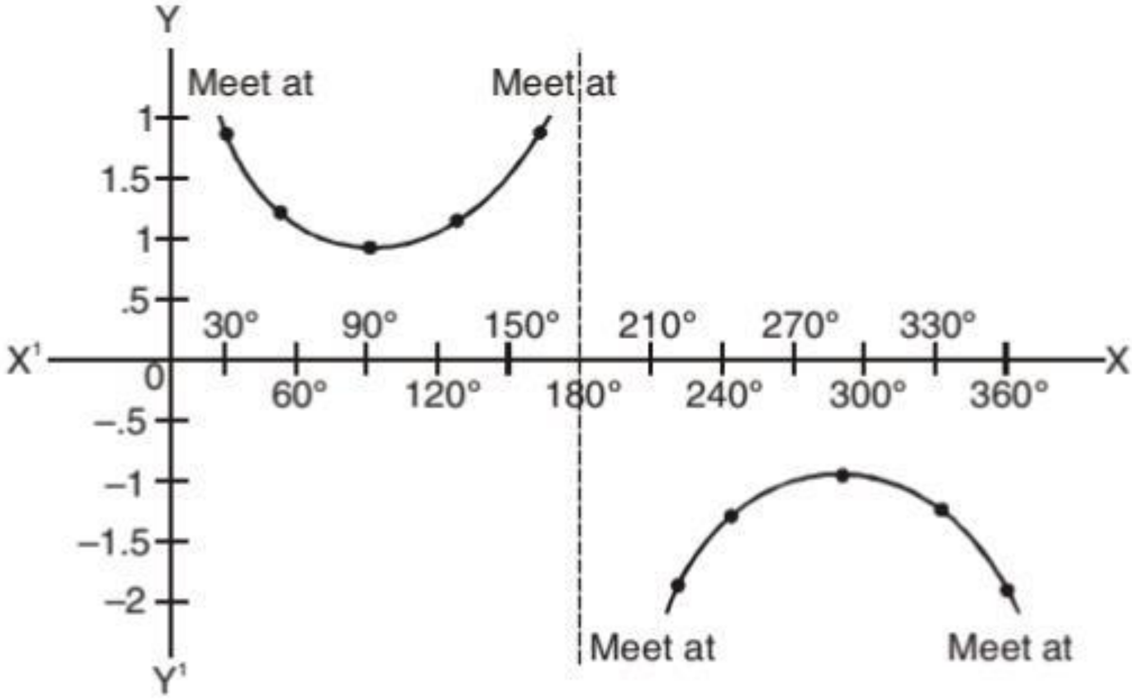

Trigonometrical Ratio of Compound Angle:

Compound Angle: An angle made up by the sum or difference of two or more angles is called compound angle.

I. Addition and Subtraction Formulae

(*i*) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

(*ii*) $\sin(A - B) = \sin A \cos B - \cos A \sin B$

(*iii*) $\cos(A + B) = \cos A \cos B - \sin A \sin B$

(*iv*) $\cos(A - B) = \cos A \cos B + \sin A \sin B$

(*v*) $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

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(*vi*) $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \tan B}$

(*vii*) $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$

(*viii*) $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$

(*ix*) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$
$= \cos^2 B - \cos^2 A$

(*x*) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$
$= \cos^2 B - \sin^2 A$

Formulae involving Double Angles:

Replacing $A = B = \theta$ in above formulae, we get

(*i*) $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

(*ii*) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$$= 2\cos^2\theta - 1 = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

(*iii*) $1 + \cos 2\theta = 2\cos^2\theta$; $1 - \cos 2\theta = 2\sin^2\theta$;

$\cos^2\theta = \dfrac{1}{2}(1+\cos 2\theta)$;

$\sin^2\theta = \dfrac{1}{2}(1-\cos 2\theta)$

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(iv) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Formulae Involving Half Angles:

Replacing 2θ by θ in above formulae, we get

(i) $\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2} = \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

(ii) $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$

$= 1 - 2\sin^2\dfrac{\theta}{2} = 2\cos^2\dfrac{\theta}{2} - 1$

$= \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

(iii) $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

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Formulae involving Triple Angles

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(ii) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(iii) $$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

(iv) $$\cot 3\theta = \frac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$$

Formulae for changing the Product into Sum or Difference.

$$\left.\begin{array}{l} 2\sin A\cos B = \sin(A+B) + \sin(A-B) \\ 2\cos A\sin B = \sin(A+B) - \sin(A-B) \end{array}\right\}, A > B$$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Formulae for changing the Sum or Difference into Product

Substituting $A + B = C$, $A - B = D$ and hence

$A = \dfrac{(C+D)}{2}$, $B = \dfrac{(C-D)}{2}$ in the above formulae, then

(i) $$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

(ii) $$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

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(iii) $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

(iv) $\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

Formulae involving Three or More Angles

(i) $\sin(\alpha+\beta+\gamma) = \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\beta\cos\gamma + \cos\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\sin\gamma$

(ii) $\cos(\alpha+\beta+\gamma) = \cos\alpha\cos\beta\cos\gamma - \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\beta\sin\gamma - \sin\alpha\sin\beta\cos\gamma$

(iii) $\tan(\alpha+\beta+\gamma)$

$$= \frac{\tan\alpha + \tan\beta + \tan\gamma - \tan\alpha\tan\beta\tan\gamma}{1-(\tan\alpha\tan\beta + \tan\beta\tan\gamma + \tan\gamma\tan\alpha)}$$

$$= \frac{\sigma_1 - \sigma_3}{1-\sigma_2}$$

(iv) $\sin(A_1 + A_2 + \ldots + A_n) = \cos A_1 \cos A_2 \ldots \cos A_n(\sigma_1 - \sigma_3 + \sigma_5 - \ldots)$

(v) $\cos(A_1 + A_2 + \ldots + A_n) = \cos A_1 \cos A_2 \ldots \cos A_n(1 - \sigma_2 + \sigma_4 - \sigma_6 + \ldots)$

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(*vi*) $\tan(A_1 + A_2 + \ldots + A_n)$

$$= \frac{\sigma_1 - \sigma_3 + \sigma_5 - \ldots}{1 - \sigma_2 + \sigma_4 - \sigma_6 + \ldots}, \text{ where}$$

$\sigma_1 = \Sigma \tan A_1$, $\sigma_2 = \Sigma \tan A_1 \tan A_2$,
$\sigma_3 = \Sigma \tan A_1 \tan A_2 \tan A_3$ etc.

Polar form and Extremas:

Polar form for $a\cos\theta + b\sin\theta$: Let $x = a\cos\theta + b\sin\theta$. Then x can be converted into cos or into sin as follows:

(*i*) Put $a = r\cos\alpha$, $b = r\sin\alpha$. Then $r = \sqrt{a^2 + b^2}$ and $\tan\alpha = b/a$.
Note that α should be evaluated by $\cos\alpha = a/r$, $\sin\alpha = b/r$. Then we get $x = r\cos(\theta - \alpha)$.

(*ii*) By putting $a = r\sin\alpha$, $b = r\cos\alpha$,

We get, $x = r\sin(\theta + \alpha)$,

$$r = \sqrt{(a^2 + b^2)} \text{ and } \cos\alpha = b/r$$

$$\sin\alpha = a/r$$

For example, Let $x = -\sqrt{3}\cos\theta + \sin\theta$

putting, $-\sqrt{3} = r\cos\alpha$, $1 = r\sin\alpha$, we get

$r = 2$, $\cos\alpha = -\sqrt{3}/2$, $\sin\alpha = 1/2$, so $\alpha = 2\pi/3$.

$\therefore x = 2\cos(\theta - 2\pi/3)$.

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Identity: A trigonometric equation is an identity if it is true for all values of the angles involved.

Conditional Identity: A conditional identity is an identity which holds if the variables satisfy a given condition. When three angles A, B, C are such that $A + B + C = 180°$ (or that A, B, C are the angles of a triangle), several identities hold between the trigonometrical function A, B, C or their multiples and sub-multiples.

Some Important Identities:

If $A + B + C = \pi$, then

(i) $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

(ii) $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(iii) $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

(iv) $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$

(v) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$

(vi) $\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$

(vii) $\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$

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(*viii*) $\cot\frac{A}{2} + \cot\frac{B}{2} + \cot\frac{C}{2} = \cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

(*ix*) $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma)$

$$= 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right)$$

(*x*) $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right)\sin\left(\frac{\gamma+\alpha}{2}\right)$$

Trigonometric Equations: Equation involving one or more than one trigonometric ratios of unknown angles are called trigonometric equations.

For example, (*i*) $2\sin\theta + \cos 2\theta = 0$

(*ii*) $3\sin^2\theta - 4\cos^2\theta = \sin\theta$ are

the trigonometric equations in unknown angle θ. A value of unknown angle satisfying the given equation is called the solution of the equation. We mainly consider the three types of equations:

(*i*) One equation in one variable

(*ii*) Two equations in one variable

(*iii*) Two equations in two variables.

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General Solution of one trigonometric equation in one variable.

1. If $\sin\theta = \sin\alpha$ or $\operatorname{cosec}\theta = \operatorname{cosec}\alpha$, then
$$\theta = n\pi + (-1)^n \alpha,\ n \in I$$
2. If $\cos\theta = \cos\alpha$ or $\sec\theta = \sec\alpha$, then
$$\theta = 2n\pi \pm \alpha,\ n \in I.$$
3. If $\tan\theta = \tan\alpha$ or $\cot\theta = \cot\alpha$, then
$$\theta = n\pi + \alpha,\ n \in I.$$
4. If $\sin^2\theta = \sin^2\alpha$ or $\cos^2\theta = \cos^2\alpha$ or $\tan^2\theta = \tan^2\alpha$ etc. then $\theta = n\pi \pm \alpha,\ n \in I$

Important Deductions:

(*i*) $\cos\theta = 0$, then $\theta = 2n\pi \pm \pi/2$ or $n\pi \pm \pi/2$ or $n\pi + \pi/2$

(*ii*) $\cos\theta = 1$, then $\theta = 2n\pi$

(*iii*) $\cos\theta = -1$, then $\theta = (2n + 1)\pi$

(*iv*) $\sin\theta = 0$, then $\theta = n\pi$

(*v*) $\sin\theta = 1$, then $\theta = 2n\pi + \pi/2$ or $n\pi + (-1)^n \pi/2$

(*vi*) $\sin\theta = -1$, then $\theta = 2n\pi - \pi/2$ or $n\pi - (-1)^n \pi/2$

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